Int J Theor Phys (2008) 47: 1600–1605
DOI 10.1007/s10773-007-9600-2

Continuous Variable Remote State Preparation with a Two-Mode Squeezed Vacuum State

E. Wu · Yong Hong Ma · Li Xia Zeng · Xiao An Zhang

Received: 29 August 2007 / Accepted: 25 October 2007 / Published online: 6 November 2007

Abstract Continuous variable remote state preparation is presented using a two-mode squeezed vacuum state based on the Heisenberg representation. We show that it is possible to remotely prepare this state in principle. Our scheme's main advantages are: (i) It is practical, because a two-mode squeezed vacuum state is easy achieved in experiment, and our scheme only requires a beam splitter and a feedforward; (ii) Using our scheme decoherence and being cheating can be avoided.

Keywords Continuous variable · Remote state preparation · Two-mode squeezed vacuum state

1 Introduction

Remote state preparation (RSP) is usually called teleporting a know state, it was firstly put forward in paper [1], where Lo has studied the classical information cost about general state preparation in RSP, using the concepts of entanglement dilution [2]. Though quantum teleportation and RSP have a similar case, they are essentially different, the main difference of them is: (1) In RSP we allow Alice to know exactly the state that she wants Bob to prepare. Particularly, Alice need not know the state, but only know the information of the state, while in teleportation the teleported state must be in Alice's side, and she need not know the target state; (2) In RSP protocol the required resource can be trade off between classical communication cost and entanglement cost. However in teleportation entanglement for per teleported qubit are not only necessary but also sufficient, and no resources can be trade off against another. Pati [3] has shown a state of a qubit chosen through equatorial or polar great

E. Wu (✉) · L.X. Zeng · X.A. Zhang
Physics Department, Xianyang Normal University, Xianyang 712000, People's Republic of China
e-mail: towue@163.com

Y.H. Ma
School of Physics and Optoelectronics, Dalian University of Technology, Dalian 116024, People's Republic of China
e-mail: myh_dlut@126.com

circles on the Block sphere can be remotely prepared by one bit from Alice to Bob when they share one ebit of entanglement. Noted that in Pati's scheme to remotely prepare a state of one qubit, the entanglement cost is the same as that in quantum teleportation [4–8], but the classical information cost is half of that in quantum teleportation, so RSP has attracted many authors [9–19]. Zeng and Zhang [9] consider the exact and minimal resource consuming RSP protocol is generalized to higher dimension, the faithful RSP using finite classical bits and a non-maximally entanglement has been studied by Ye, Zhang and Guo [14]. Berry and Sanders [20] have discussed remote preparation of ensemble of mixed states.

All above protocol mentioned so far deal with finite dimensional quantum systems. Nevertheless, for quantum communication light pulses are essentially described by continuous variables (CV) and CV quantum information provides an interesting alternative to the traditional qubit-based technique [21]. For CV teleportation [22, 23], the nonlocal resource shared by Alice and Bob is the EPR state with preferable correlation in position and momentum, In quantum optics, this correlation is approximate with a highly squeezed two-mode state of Electromagnetic field with quadrature amplitudes of position and momentum. At Alice's site Bell measurement is changed to the measurement of the position $\hat{X} \equiv \hat{X}_1 + \hat{X}_{\text{in}}$ and the momentum $\hat{P} \equiv \hat{P}_1 - \hat{P}_{\text{in}}$ of Alice's half of the EPR pair and input particle. Then Bob implements the unitary operations of the phase-space displacement $\hat{D}(\alpha)$, where $\alpha = X + iP$ are obtained by the measurement result. Though CV schemes play an important role in quantum information [24], comprehensive studding continuous variable remote state preparation is still few, Kurucz et al. [25] extend exact deterministic RSP with minimal classical communication to continuous variable quantum system. We will thoroughly discuss CVRSP using a two-mode squeezed vacuum state from the point of decreasing classical resources and avoiding decoherence. The paper organized as follows, in Sect. 2, we present our scheme about continuous variable remote state preparation using a two-mode squeezed vacuum state; In Sect. 3, we discuss to our scheme's superiority. The conclusion is drawn in Sect. 4.

2 RSP Scheme with Continuous Variables

Firstly we describe this scheme based on the Heisenberg representation. Consider a two-mode squeezed vacuum state which can be produced by the unitary two-mode squeezed operator [24]

$$\hat{U}(t,0) \equiv \hat{S}(\xi) = \exp(\xi^* \hat{a}_1 \hat{a}_2 - \xi \hat{a}_1^+ \hat{a}_2^+), \tag{1}$$

where $\hat{a}_1, \hat{a}_2$ and $\hat{a}_1^+, \hat{a}_2^+$ are the creation and annihilation operators respectively, ξ is squeezed parameter, and corresponding to the non-degenerate optical parametric amplifier interaction Hamiltonian $\hat{H}_{\text{in}} = i\hbar\kappa(\hat{a}_1^+ \hat{a}_2^+ e^{i\theta} - \hat{a}_1 \hat{a}_2 e^{-i\theta})$ is equivalent to the two-mode state emerging from a 50:50 beam splitter with two single-mode squeezed vacuum state at the input. So the quadrature operators of the two-mode squeezed vacuum state can be written as

$$\hat{X}_1 = (e^{+r} \hat{X}_1^{(0)} + e^{-r} \hat{X}_2^{(0)})/\sqrt{2}, \tag{2}$$

$$\hat{P}_1 = (e^{-r} \hat{P}_1^{(0)} + e^{+r} \hat{P}_2^{(0)})/\sqrt{2}, \tag{3}$$

$$\hat{X}_2 = (e^{+r} \hat{X}_1^{(0)} - e^{-r} \hat{X}_2^{(0)})/\sqrt{2}, \tag{4}$$

$$\hat{P}_2 = (e^{-r} \hat{P}_1^{(0)} - e^{+r} \hat{P}_2^{(0)})/\sqrt{2}, \tag{5}$$

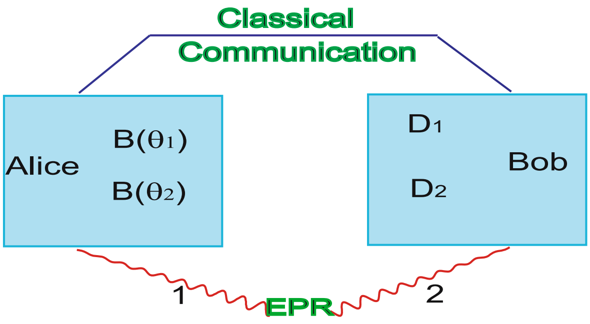

Fig. 1 Teleportation of a single mode of the electromagnetic field. Alice and Bob shared the entanglement of mode 1 and mode 2, and a classical communication channel. Alice know the state she wants Bob to prepare according to remote preparation $\hat{X}_{\text{out}}$. Alice performs a unitary operation $\hat{B}(\theta_1)$ and $\hat{B}(\theta_2)$. then Alice can yield classical results, Bob perform displacement $\hat{D}_1$ and $\hat{D}_2$ that is dependent of classical results

thus the relative position and the total momentum are

$$\hat{X}_1 - \hat{X}_2 = \sqrt{2}e^{-r}\hat{X}_2^{(0)}, \tag{6}$$

$$\hat{P}_1 + \hat{P}_2 = \sqrt{2}e^{-r}\hat{P}_1^{(0)}. \tag{7}$$

A superscript "(0)" denotes initial vacuum modes, and "r" is the squeezing parameters, which is fixed in the protocol. In (6, 7) mode 1 and 2 are entangled to a finite degree, corresponding to a non-maximally entangled state. Now mode 1 is sent to Alice called the sender, mode 2 is sent to Bob called the receiver (Fig. 1), suppose Alice wants Bob to prepare modes

$$\hat{X}_{\text{out}} = \sqrt{2}e^{-r}\hat{X}_0, \tag{8}$$

$$\hat{P}_{\text{out}} = \sqrt{2}e^{-r}\hat{P}_0. \tag{9}$$

This prepared modes are know to Alice, she perform a unitary operation $\hat{B}_\theta$ with a ideal beam splitter according to remote preparation. We define the action of an ideal phase-free beam splitter operation on a pair of modes [26], after operation of the ideal beam splitter modes $\hat{X}_1$, $\hat{P}_1$ become

$$\begin{aligned}\hat{X}_1' &= \hat{B}(\theta_1)(\hat{X}_1)\\ &= \hat{B}(\theta_1)\left(\frac{1}{\sqrt{2}}e^{+r}\hat{X}_1^{(0)} + \frac{1}{\sqrt{2}}e^{-r}\hat{X}_2^{(0)}\right)\\ &= \frac{1}{\sqrt{2}}e^{+r}\hat{X}_1^{(0)}\cos\theta_1 + \frac{1}{\sqrt{2}}e^{-r}\hat{X}_2^{(0)}\sin\theta_1,\end{aligned} \tag{10}$$

$$\begin{aligned}\hat{P}_1' &= \hat{B}(\theta_2)(\hat{P}_1)\\ &= \hat{B}(\theta_2)\left(\frac{1}{\sqrt{2}}e^{-r}\hat{P}_1^{(0)} + \frac{1}{\sqrt{2}}e^{+r}\hat{P}_2^{(0)}\right)\\ &= \frac{1}{\sqrt{2}}e^{-r}\hat{P}_1^{(0)}\cos\theta_2 + \frac{1}{\sqrt{2}}e^{+r}\hat{P}_2^{(0)}\sin\theta_2.\end{aligned} \tag{11}$$

We can regulate the angle θ to $\theta_1 = \frac{\pi}{2}, \theta_2 = 0$, and derive

$$\hat{X}_1' = \frac{1}{\sqrt{2}} e^{-r} \hat{X}_2^{(0)}, \tag{12}$$

$$\hat{P}_1' = \frac{1}{\sqrt{2}} e^{-r} \hat{P}_1^{(0)}. \tag{13}$$

$\hat{B}(\theta)$ is under Alice's control but is unknown to Bob, so this transforms the entanglement into

$$\frac{1}{\sqrt{2}} e^{-r} \hat{X}_2^{(0)} - \hat{X}_2 = \sqrt{2} e^{-r} \hat{X}_2^{(0)}, \tag{14}$$

$$\frac{1}{\sqrt{2}} e^{-r} \hat{P}_1^{(0)} - \hat{P}_2 = \sqrt{2} e^{-r} \hat{P}_1^{(0)}. \tag{15}$$

Rewrite the equation, we can get

$$\hat{X}_2 = -\frac{\sqrt{2}}{2} e^{-r} \hat{X}_2^{(0)}, \tag{16}$$

$$\hat{P}_2 = \frac{\sqrt{2}}{2} e^{-r} \hat{P}_1^{(0)}. \tag{17}$$

Now we define the displacement operator

$$\hat{D}(\hat{X}_1)\hat{X}_2 = \hat{X}_1 + \hat{X}_2. \tag{18}$$

After received Alice's measurement result, Bob will perform the corresponding phase-space displacement operation $\hat{D}(-2\hat{X}_0 - \hat{X}_2^{(0)})$ and $\hat{D}(2\hat{P}_0 - \hat{P}_1^{(0)})$ to $\hat{X}_2$, $\hat{P}_2$ respectively, such phase-space displacements can be easy performed for the continuous quadrature amplitudes using feedforward techniques.

$$\begin{aligned} \hat{D}(-2\hat{X}_0 - \hat{X}_2^{(0)})(\hat{X}_2) &= -\frac{\sqrt{2}}{2} e^{-r} \hat{D}(-2\hat{X}_0 - \hat{X}_2^{(0)})(\hat{X}_2^{(0)}) \\ &= -\frac{\sqrt{2}}{2} e^{-r} (-2\hat{X}_0 - \hat{X}_2^{(0)} + \hat{X}_2^{(0)}) \\ &= \sqrt{2} e^{-r} \hat{X}_0, \end{aligned} \tag{19}$$

$$\begin{aligned} \hat{D}(2\hat{P}_0 - \hat{P}_1^{(0)})(\hat{P}_2) &= \frac{\sqrt{2}}{2} e^{-r} \hat{D}(2\hat{P}_0 - \hat{P}_1^{(0)})(\hat{P}_1^{(0)}) \\ &= \frac{\sqrt{2}}{2} e^{-r} (-2\hat{P}_0 - \hat{P}_1^{(0)} + \hat{P}_1^{(0)}) \\ &= \sqrt{2} e^{-r} \hat{P}_0. \end{aligned} \tag{20}$$

Above results are just about the target state Alice wants Bob to prepare. In brief, in order to gain target state the scheme consists of three steps: (i) Set up entanglement state between Alice and Bob; (ii) Alice perform a unitary operation with a beam splitter on her half of the shared entanglement according to the state that she wants to prepare remotely at Bob's site and inform its result to Bob; (iii) Bob applies a displacement operation on his half according to Alice's information to restore the target state.

3 Discussion to the Result Contrast to Teleportation

1. Reduce Classical Resources in Contrast to Quantum Teleportation For the sake of clarity, we give the quantum teleportation scheme as follow, Alice and Bob shared the entanglement (6) and (7). Now mode 1 is sent to Alice and mode 2 is sent to Bob, Alice's mode is then combined at a 50:50 beam splitter with the input mode "in". The homodyne detectors D^x and D^p yield classical photocurrents for the quadratures x_u and p_v

$$\hat{X}_u = \frac{1}{\sqrt{2}}(\hat{X}_{\text{in}} - \hat{X}_1), \tag{21}$$

$$\hat{X}_v = \frac{1}{\sqrt{2}}(\hat{X}_{\text{in}} + \hat{X}_1), \tag{22}$$

$$\hat{P}_u = \frac{1}{\sqrt{2}}(\hat{P}_{\text{in}} - \hat{P}_1), \tag{23}$$

$$\hat{P}_v = \frac{1}{\sqrt{2}}(\hat{P}_{\text{in}} + \hat{P}_1). \tag{24}$$

Using above equations and (6, 7), we may write Bob's mode 2 as

$$\begin{aligned}\hat{X}_2 &= \hat{X}_{\text{in}} - (\hat{X}_1 - \hat{X}_2) - \sqrt{2}\hat{X}_u \\ &= \hat{X}_{\text{in}} - \sqrt{2}e^{-r}\hat{X}_2^{(0)} - \sqrt{2}\hat{X}_u,\end{aligned} \tag{25}$$

$$\begin{aligned}\hat{P}_2 &= \hat{P}_{\text{in}} + (\hat{P}_1 + \hat{P}_2) - \sqrt{2}\hat{P}_v \\ &= \hat{P}_{\text{in}} + \sqrt{2}e^{-r}\hat{P}_1^{(0)} - \sqrt{2}\hat{P}_v.\end{aligned} \tag{26}$$

Alice's Bell detection yields certain classical values x_u and p_v for $\hat{X}_u$ and $\hat{P}_v$. The quantum variables $\hat{X}_u$ and $\hat{P}_v$ become classically determined, random variables x_u and p_v. After receiving Alice's classical result x_u and p_v, Bob displaces his mode, thus we obtain

$$\hat{X}_{\text{out}} \equiv \hat{X}_{\text{in}} = \hat{X}_2 + \sqrt{2}e^{-r}\hat{X}_2^{(0)} + \sqrt{2}\hat{X}_u, \tag{27}$$

$$\hat{P}_{\text{out}} \equiv \hat{P}_{\text{in}} = \hat{P}_2 - \sqrt{2}e^{-r}\hat{P}_1^{(0)} + \sqrt{2}\hat{P}_v. \tag{28}$$

We can clearly see, in RSP protocol Alice only message Bob to perform the operation $D(-2\hat{X}_0 - \hat{X}_2^{(0)})$ and $D(2\hat{P}_0 - \hat{P}_1^{(0)})$, but in teleportation protocol Alice need notice Bob $\hat{X}_2^{(0)}$, $\hat{X}_u$, $\hat{P}_1^{(0)}$, $\hat{P}_v$, evidently, teleportation needs more classical message than RSP.

2. Avoid Being Cheating and Decoherence The teleportation scheme with Alice and Bob is completely without any further measurement, the teleportation state remains unknown to both Alice and Bob, and need not be demolished in a detection by Bob as a final step. However, maybe Alice and Bob are easy cheating. Suppose that instead of using an EPR channel they try to get away without entanglement and use only a classical channel, for a realistic experimental situation with finite squeezing and inefficient detectors where perfect teleportation is unattainable. In contrast, in RSP Alice knows the mode that needs to be transferred, other operation is completed by Bob, this can avoid being cheating.

In addition, in teleportation for EPR correlations between the two modes become ideal. We must limit the squeezing parameter $r \to \infty$. But the individual modes become very noisy, decoherence may rise, however, teleportation in RSP scheme squeezing parameter r is arbitrary.

4 Summary

We have presented continuous variable remote state preparation protocol in Heisenberg representation making use of a two-mode squeezed state. The possibility of our scheme is demonstrated in principle, the main reason is in the continuous variable, not only the generation of continuous variable entanglement, but also its manipulation via measurements and unitary operations, turn out to be very easy. Our scheme only requires a beam splitter and a feedforward. We also show it has two advantages: (i) It is practical because a two-mode squeezed vacuum state is easy achieved in experiment; (ii) Using our scheme decoherence and being cheating can be avoided.

References

1. Lo, H.K.: Phys. Rev. A **62**, 012313 (2000)
2. Bennett, C.H., et al.: Phys. Rev. A **53**, 2046 (1996)
3. Pati, A.K.: Phys. Rev. A **63**, 014302 (2001)
4. Bennett, C.H., Brasard, G., Crepeall, C., Jozsa, R., Peres, S., Wooters, W.K.: Phys. Rev. Lett. **70**, 1895 (1993)
5. Bouwmeester, D., Pan, J.W., Mattle, K., Eible, M., Weinfurter, H., Zeilinger, A.: Nature (London) **390**, 575 (1997)
6. Boschi, D., et al.: Phys. Rev. Lett. **80**, 1121 (1998)
7. Furusawa, A., et al.: Science **282**, 706 (1998)
8. Kim, Y.H., Kulik, S.P., Shih, Y.H.: Phys. Rev. Lett. **86**, 1370 (2001)
9. Zeng, B., Zhang, P.: Phys. Rev. A **65**, 022316 (2002)
10. Devetak, I., Berger, T.: Phys. Rev. Lett. **87**, 197901 (2001)
11. Leung, D.W., Shor, P.W.: Phys. Rev. Lett. **90**, 127905 (2003)
12. Hayashi, A., Hashimoto, T., Horibe, M.: Phys. Rev. A **67**, 052302 (2003)
13. Abeyesinghe, A., Hayden, P.: Phys. Rev. A **68**, 062319 (2003)
14. Ye, M.Y., Zhang, Y.S., Guo, G.C.: Phys. Rev. A **69**, 022310 (2004)
15. Kurecz, Z., Adam, P.: J. Opt. B: Quantum Semiclass. Opt. 7135 (2005)
16. Yu, C.S., Song, H.S., Wang, Y.H.: Phys. Rev. A **73**, 022340 (2006)
17. Rosenfeld, W., Berner, S., Volz, J., Weber, M., Weinfurter, H.: Phys. Rev. Lett. **98**, 050504 (2007)
18. Xia, Y., Song, J., Song, H.S.: J. Phys. B **40**, 3719 (2007)
19. Xia, Y., Song, J., Song, H.S.: Opt. Commun. **277**, 219 (2007)
20. Berry, D.W., Sanders, B.C.: Phys. Rev. Lett. **90**, 057901 (2003)
21. Lloyd, S., Braunstein, S.L.: Phys. Rev. Lett. **82**, 1784 (1999)
22. Vaidman, L.: Phys. Rev. A **49**, 1473 (1994)
23. Braunstein, S.L., Kimble, H.J.: Phys. Rev. Lett. **80**, 869 (1998)
24. Braunstein, S.L.: Rev. Modern Phys. **77**, 513–577 (2005)
25. Kurucz, Z., Adam, P., Kis, Z., Janszky, J.: Phys. Rev. A **72**, 052315 (2005)
26. Van Loock, P., Braunstein, S.L.: Phys. Rev. Lett. **84**, 3482 (2000)